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Connectivity in sensor networks

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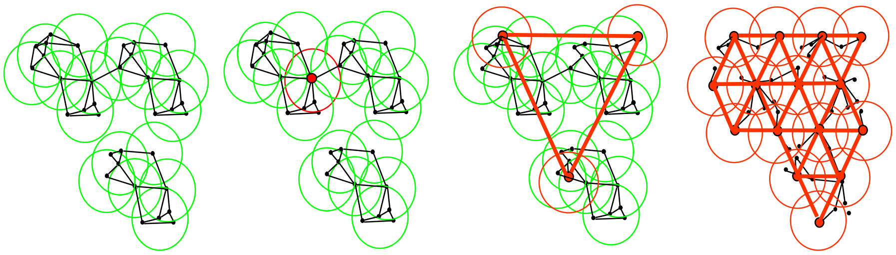
Ad-hoc <-> sensor <-> hybrid <-> cellular



- ☐ Connectivity is an essential issue in wireless
  - ad hoc networks (many to many, global)
  - sensor networks (many to one, global)
  - hybrid (multi-hop cellular) networks (many to many, local)
- ☐ Position of nodes is a (often homogeneous Poisson) spatial point process, with intensity λ.
- ☐ Position of base stations at nodes of a honey-comb grid, all connected to each other by a wired network.

The Boolean model with circular grains

☐ Fixed radio range r

☐ Nodes i and j, at positions x_i and x_j are directly connected iff

$$\left\| x_i - x_j \right\| < r$$

☐ References:

- E. N. Gilbert, « Random plane networks », SIAM Journal, 1961.
- R. Meester and R. Roy, « Continuum percolation », Cambridge University Press, 1996.
- O. Dousse, P. Thiran, M. Hasler, « Connectivity in ad hoc and hybrid networks », Infocom 2002.

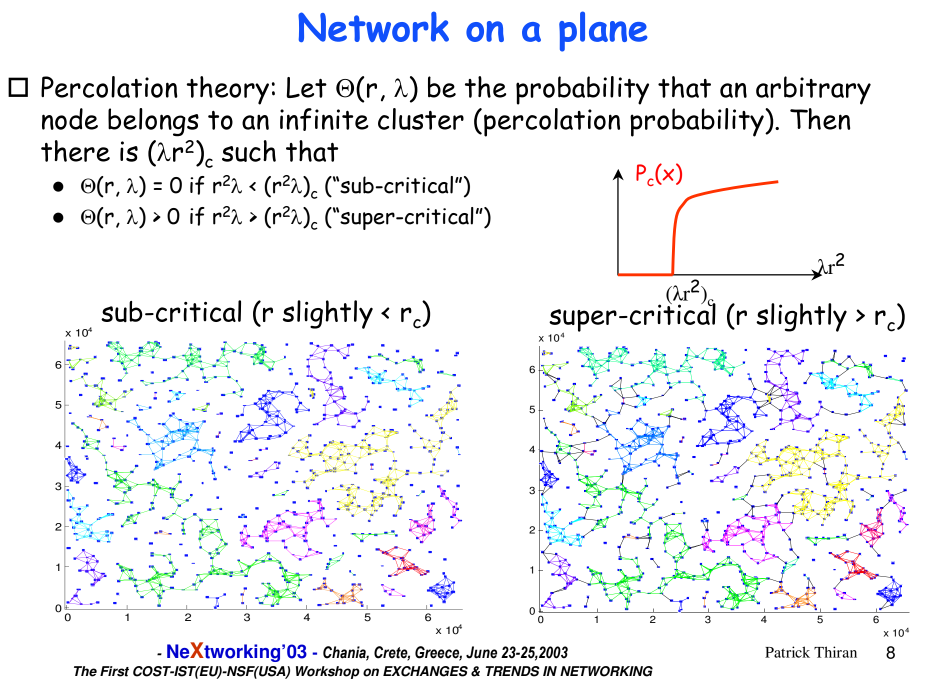
Connectivity in packet radio networks

☐ Finite domain: is the network fully connected ?

- Kleinrock & Silvester (1978) « Optimum transmission radii in packet radio networks or why six is a magic number » : $\pi r^2 \lambda = 5.89$ is a good value for throughput
- Philips, Panwar, Tantawi (1989) « Connectivity properties of a packet radio network » : $\pi r^2 \lambda$ must grow logarithmically with the area of the domain
- Gupta & Kumar (1998) « Critical power for asymptotic connectivity in wireless networks » : for $\lambda \to \infty$, $\pi r^2 \lambda = \log \lambda + K(\lambda)$ where $K(\lambda) \to \infty$.
- Shakkottai, Srikant, Shroff (2003) « Unreliable sensor grids: Coverage, Connectivity and Diameter»: $\pi r^2 \lambda \approx \log \lambda / p(\lambda)$ where $p(\lambda)$ is the node failure prob.

☐ Infinite domain: is there an infinite connected component (percolation theory):

- Continuum percolation: there exist a finite $(\pi r^2 \lambda)_c$, below which all connected components are a.s. bounded and above which there is a unique infinite connected component (Gilbert (1961), Hall (1985), Zuev & Siderenko (1985), Menshikov (1986), Meester & Roy (1990, 1994))
- $1.64 < (\pi r^2 \lambda)_c < 17.9$ (Gilbert (1961))
- $2.195 < (\pi r^2 \lambda)_c < 10.526$ (Philips, Panwar, Tantawi (1989))
- $(\pi r^2 \lambda)_c \approx 4.5$ (numerical value, Quintanilla, Torquato, Ziff (2000))



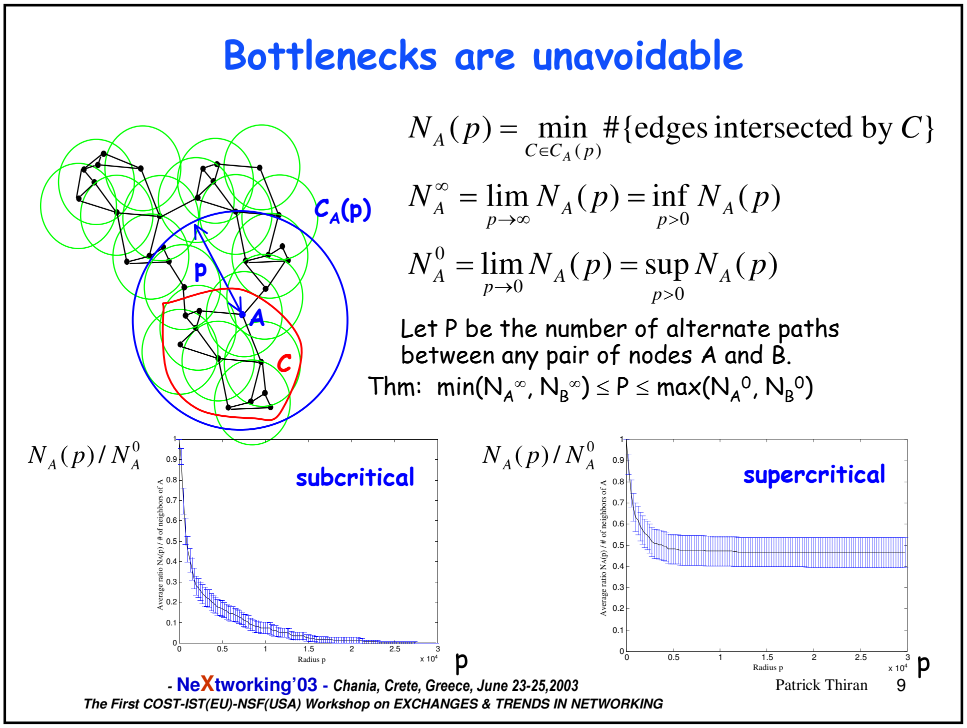
Network on a plane

☐ Percolation theory: Let Θ(r, λ) be the probability that an arbitrary node belongs to an infinite cluster (percolation probability). Then there is $(\lambda r^2)_c$ such that

- $\Theta(r, \lambda) = 0$ if $r^2\lambda < (r^2\lambda)_c$ ("sub-critical")
- $\Theta(r, \lambda) > 0$ if $r^2\lambda > (r^2\lambda)_c$ ("super-critical")

sub-critical (r slightly < r_c)

super-critical (r slightly > r_c)



Bottlenecks are unavoidable

$$N_A(p) = \min_{C \in C_A(p)} \#\{\text{edges intersected by } C\}$$

$$N_A^\infty = \lim_{p\to\infty} N_A(p) = \inf_{p>0} N_A(p)$$

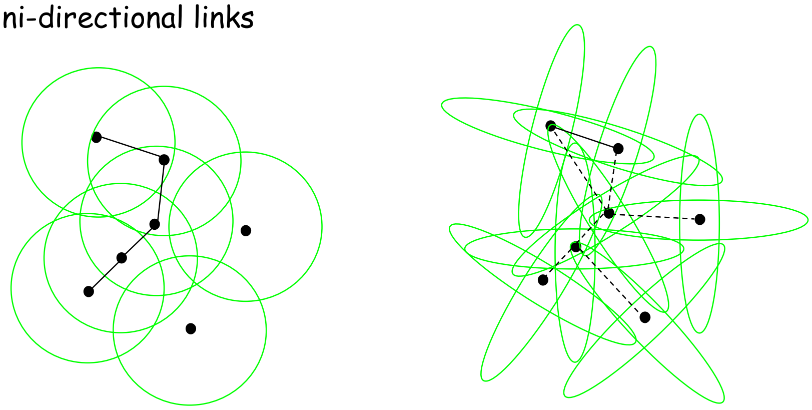
$$N_A^0 = \lim_{p\to 0} N_A(p) = \sup_{p>0} N_A(p)$$

Let P be the number of alternate paths between any pair of nodes A and B.

Thm: $\min(N_A^\infty, N_B^\infty) \le P \le \max(N_A^0, N_B^0)$

Beyond the Boolean model with circular grains: irregularity helps

- ☐ Percolation occurs sooner for elongated shapes (Penrose (1993), Booth, Bruck, Cook, Franceschetti (2003))
- ☐ Possible advantage of directional antennas
- ☐ Uni-directional links



Beyond the Boolean model: the physical (STIRG) model

- ☐ Signal to Noise Ratio at Node j receiving from Node i is

$$SNR_{i\to j} = \frac{PL\left(\|x_i - x_k\|\right)}{N_0 + \gamma \sum_{k\neq i,j} PL\left(\|x_i - x_k\|\right)}$$

  - P = Emitting power
  - L(d) = Attenuation function at distance d (e.g., L(d) = $d^{-\alpha}$)
  - N_0 = Background thermal noise
  - γ = degree of orthogonality of the code (γ = 1 for a narrowband system, $0 \leq \gamma < 1$ for a CDMA system)

- ☐ Nodes i and j are directly connected iff

$$\min\{SNR_{i\to j}, SNR_{j\to i}\} > \beta$$

- ☐ Reference:
  O. Dousse, F. Baccelli, P. Thiran, « Impact of Interferences on Connectivity in ad hoc networks », Infocom 2003.